The Master Product Method for Factoring Quadratics

The Standard Format for a quadratic polynomial is

$$\mathbf{A}x^2 + \mathbf{B}x + \mathbf{C}$$

In order to use the Master Product Method, you must first write your quadratic in this Standard Format.

It is important to notice that this form requires that all arithmetic operators are "+" (addition). If the original quadratic has subtraction as an operator, change it to addition and apply the negative sign to the term following that operator.

Find the product of **A** (coefficient of the x^2-term) and **C** (the constant-term). This is your "**master product**".

Remember that the signs of the numbers **A, B** & **C** matter. So, does the resulting sign of the master product.

Next, find all possible pairs of integral factors of the master product. *Hint:* If the master product is positive, either both factors are positive or both are negative. If the master product is negative, exactly one of the factors must be negative.

Select the pair of factors for the master product whose sum is **B** (the coefficient of the x-term). *If there is no such pair of factors, you cannot factor this quadratic.*

Rewrite your quadratic polynomial by replacing the original x-term with an equivalent expression that has two x-terms. Create this expression by using each of the integers from the pair of factors you selected as the coefficients of two new x-terms & add these new terms together.

Next, regroup the terms of your polynomial so that you can factor the grouped pairs of terms to create a common factor from each group.

Use the Distributive Law of Multiplication Over Addition to rewrite the expression as two binomial factors. Your polynomial has been factored!

To check your answer, multiply out these binomial factors. You should get your original quadratic polynomial as the result.

EXAMPLE

$6x^2 - 5x - 6$

$\mathbf{A}x^2 + \mathbf{B}x + \mathbf{C} =$

$6x^2 + (-5x) + (-6)$

So, $\mathbf{A} = +6$, $\mathbf{B} = -5$, $\mathbf{C} = -6$

$MP = (\mathbf{A})(\mathbf{C}) = (+6)(-6) = -36$

+1 & -36
-1 & +36
+2 & -18
-2 & +18
+3 & -12
-3 & +12
+4 & -9
-4 & +9
-6 & +6

B = -5, so we pick +4 & -9 because (+4) + (-9) = -5

$6x^2 + (-5x) + (-6) =$

$6x^2 + [(+4x) + (-9x)] + (-6)$

=

$[6x^2 + (+4x)] + [(-9x) + (-6)]$

=

$(2x)\{3x + 2\} + (-3)\{3x + 2\} =$

$(2x - 3)\{3x + 2\}$

or

$(2x - 3)(3x + 2)$

Factoring using "Bottoms Up" method.

Always put your polynomial in descending order.
Next, always factor out the greatest common factor (if there is one)

Example: $40x^2+76x+24$

Notice: GCF (greatest common factor): 4

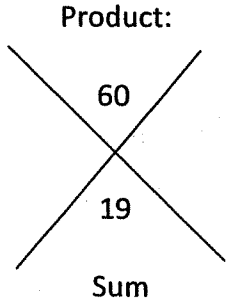
Factor out a 4:

$4(10x^2+19x+6)$

Now we can begin the bottoms up method on the trinomial: $10x^2+19x+6$

The method begins similarly to the Master Product/AC Method.

You find the product of the leading coefficient with the ending term:
$10\cdot 6=60$
Now make note of the middle coefficient: 19.

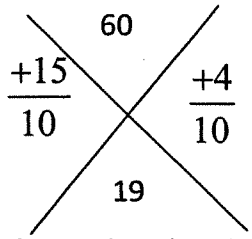


I like to organize them in a big X:

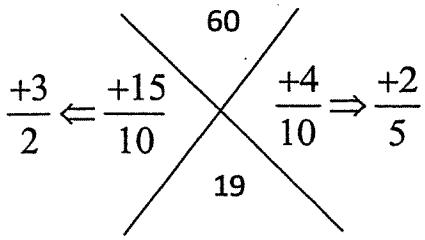
Next, find two numbers that multiply to +60
and at the same time add up to +19.

Put those in your diagram (on the left and right) when you find them.
Use your knowledge of signed numbers to know that both numbers (in this case) must be positive: +15 and +4.

Insert those numbers on the sides.
Next divide those numbers by
your leading coefficient: 10



You must simplify the two fractions (always keep the sign in the numerator).



$\frac{+3}{2}\rightarrow(2x+3)$

$\frac{+2}{5}\rightarrow(5x+2)$

Now we bring the " bottom up" (to the left) and it will become the leading coefficient in each of your two binomials. As seen to the right above.

Solution: $40x^2+76x+24=4(5x+2)(2x+3)$

Note: don't forget your GCF of 4!